ES1004 Econometrics by Example

Lecture 9: Stationary and Non-Stationary Time Series

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Gujarati textbook, second edition [chapter 13]

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Time Series Econometrics

13. stationary and nonstationary time series
14. cointegration and error correction models
15. asset price volatility: the ARCH and GARCH models
16. economic forecasting

- previous course on time series econometrics

ES1002 Lectures

ES1002 EViews

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Introduction

- regression analysis of time series data assumes that series are stationarity
  - its mean and variance are constant over time
  - covariance depends only on the distance between the two periods and not on time
  - a time series with these characteristics is know as weakly or covariance stationary

strictly stationary

- a time series is an example of a stochastic process [a sequence of random variables ordered in time]

Example: Exchange Rate

- data `table13_1.xls`
  - the exchange rate between the us dollar and the euro EX; dollars per unit of euro
  - daily from January 4, 2000 to May 8,2008 [2355 observations]
  - are not continuous; exchange rate markets are not always open every day and because of holidays
  - see figure next slide

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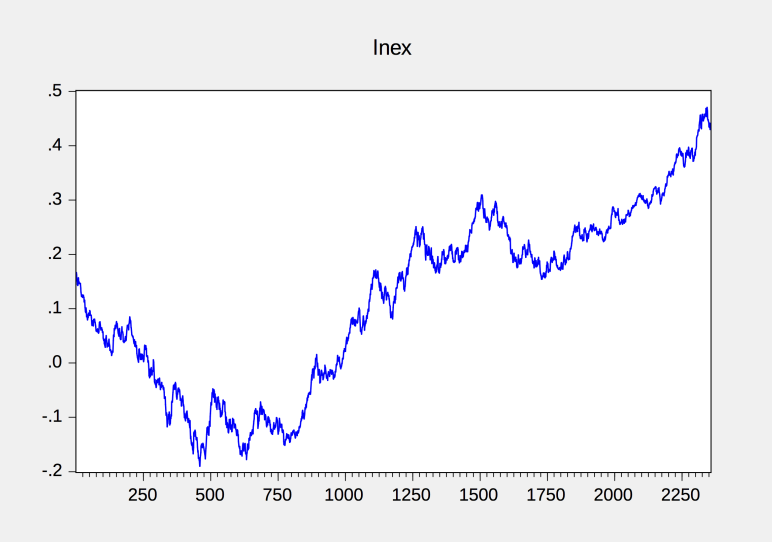
Exchange Rate



Importance of Stationary Time Series

- most economic time series in level form are nonstationary
  - such series often exhibit an upward or downward trends over a sustained period of time
  - but such a trend is often stochastic and not deterministic
- regressing a nonstationary time series on one or more nonstationary time series may often lead to spurious [meaningless] regression
  - high R^2, statistically significant regression coefficients, but not reliable

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Diagnostic Tools

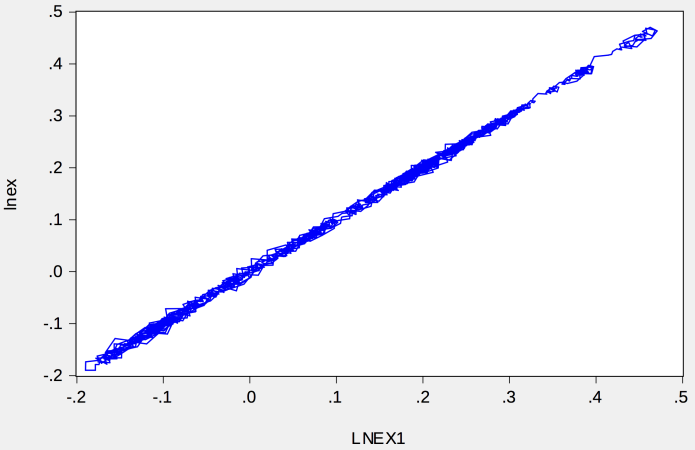
- it is important to find out if a time series is stationary
- three ways to examine the stationary of a time series
  1. graphical analysis
  2. correlogram
  3. unit root tests

Graphical Analysis

"anyone who tries to analyse analyse a time series without plotting it first is asking for trouble"

Chatfield (2004)

Graphical Analysis



Autocorrelation Function ACF

- shows if the correlation of the time series over several lags decays quickly or slowly
    - if it does decay very slowly, perhaps the time series is nonstationary
- the ACF at lag k is defined as covariance at lag k divided by variance

$$\rho_k = \frac{\gamma_k}{\gamma_0} \tag{1}$$

- akaike or schwarz information criterion to determine the lag length; or
- compute ACF up to one-quarter to one-third of series length

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Correlogram I

- a plot of $\hat{\rho}_k$ against k, the lag length, the (sample) correlogram
- test the statistical significance of each autocorrelation coefficient in the correlogram by
  - computing its standard error; or
  - find out if the sum of autocorrelation coefficients is statistically significant (distributed as chi-square) using the Q statistic, where n is the sample size and m is the number of lags (=df)

$$Q = n \sum_{k=1}^{k=m} \hat{\rho}_k^2 \tag{2}$$

Correlogram II

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.998	0.998	2350.9	0.000
		2	0.997	0.004	4695.7	0.000
		3	0.995	-0.017	7034.2	0.000
		4	0.994	0.012	9366.6	0.000
		5	0.992	-0.014	11693.	0.000
		6	0.991	0.012	14013.	0.000
		7	0.989	-0.020	16326.	0.000
		8	0.988	-0.018	18633.	0.000
		9	0.986	0.006	20934.	0.000
		10	0.984	0.001	23228.	0.000
		11	0.983	0.001	25516.	0.000
		12	0.981	-0.024	27796.	0.000
		13	0.979	-0.019	30070.	0.000
		14	0.978	-0.001	32337.	0.000
		15	0.976	0.016	34597.	0.000
		16	0.974	-0.007	36850.	0.000
		17	0.973	-0.010	39097.	0.000
		18	0.971	0.020	41336.	0.000

Unit Root Test I

- the unit root test for the exchange rate can be expressed as follows

$$\Delta LEX_t = \beta_1 + \beta_2 t + \beta_3 LEX_{t-1} + u_t \tag{3}$$

- we regress the first differences of the log of exchange rate on the trend variable and the one-period lagged value of the exchange rate
- the term unit root

Unit Root Test II

- the null hypothesis is that β_3, the coeffcieint of LEX_{t-1}, is zero
  - the unit root hypothesis
- the alternative hypothesis is $\beta_3 < 0$
- a non-rejection of the null hypothesis would suggest that the time series under consideration is nonstationary
- cannot use t test because it assumes stationarity

Dickey-Fuller Test

- dickey and fuller developed a τ [tau] test; critical values are calculated by simulations
- estimate Eq. 3 by OLS and look at the routinely calculated t value of $\hat{\beta}_3$ but use the DF critical values
  - reject the null if computed t $(= \tau)$ [in absolute value] exceeds the DF critical value
  - consider the absolute value as the coefficient β_3 is expected be negative

Dickey-Fuller Test: Example

Dependent Variable: DLNEX
Method: Least Squares
Date: 08/27/16 Time: 13:20
Sample (adjusted): 2 2355
Included observations: 2354 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000847	0.000292	-2.899171	0.0038
TIME	1.21E-06	3.22E-07	3.761595	0.0002
LNEX(-1)	-0.004088	0.001351	-3.026490	0.0025

R-squared	0.005995	Mean dependent var	0.000113
Adjusted R-squared	0.005149	S.D. dependent var	0.005926
S.E. of regression	0.005911	Akaike info criterion	-7.422695
Sum squared resid	0.082147	Schwarz criterion	-7.415349
Log likelihood	8739.512	Hannan-Quinn criter.	-7.420020
F-statistic	7.089626	Durbin-Watson stat	1.999138
Prob(F-statistic)	0.000852		

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Dickey-Fuller Test: Practical Aspects

- the DF test can be performed in three different forms

1. random walk
   $\Delta LEX_t = \beta_3 LEX_{t-1} + u_t$
2. random walk with drift
   $\Delta LEX_t = \beta_1 + \beta_3 LEX_{t-1} + u_t$
3. random walk with drift around deterministic trend
   $\Delta LEX_t = \beta_1 + \beta_2 t + \beta_3 LEX_{t-1} + u_t$

- same null hypothesis but the critical DF values are different
- guard against model specification errors

Augmented Dickey-Fuller Test

- if the error term u_t is correlated, use the ADF test
- add the lagged values of the dependent variable as follows

$$\Delta LEX_t = \beta_1 + \beta_2 t + \beta_3 LEX_{t-1} + \sum_{t=1}^{m} \alpha_i \Delta LEX_{t-1} + \epsilon_t \tag{4}$$

- ϵ_t is a pure white noise error term

white noise

- m is the maximum length of the lagged dependent variable

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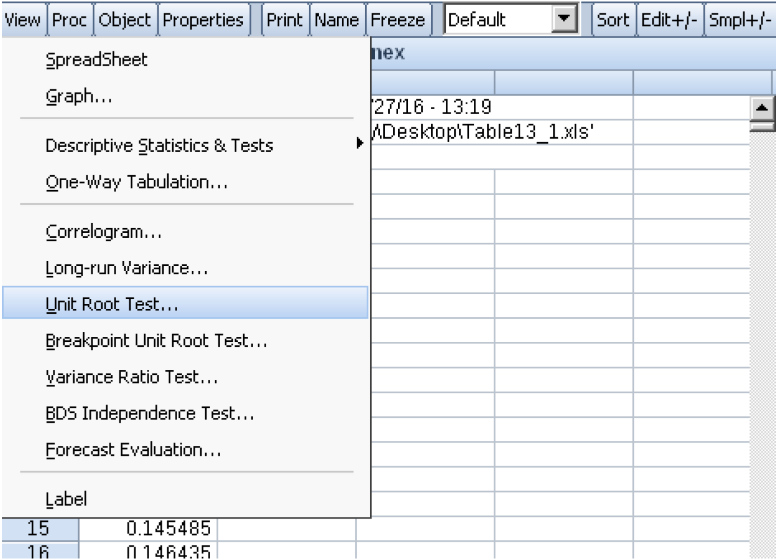
ADF: EViews I


View Proc Object Properties Print Name Freeze Default Sort Edit+/- Smpl+/-
SpreadSheet
Graph...
Descriptive Statistics & Tests
One-Way Tabulation...
Correlogram...
Long-run Variance...
Unit Root Test...
Breakpoint Unit Root Test...
Variance Ratio Test...
BDS Independence Test...
Forecast Evaluation...
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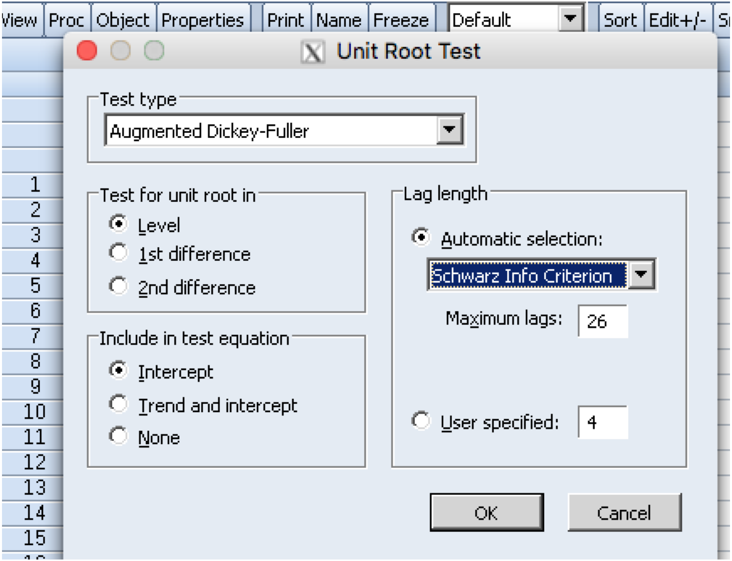
\Desktop\Table13_1.xls'
15 0.145485
16 0.146435

ADF: EViews II



ADF: Intercept I

Null Hypothesis: LNEX has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=26)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		0.171718	0.9708
Test critical values:	1% level	-3.432933	
	5% level	-2.862567	
	10% level	-2.567362	

*MacKinnon (1996) one-sided p-values.

ADF: Intercept II

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(LNEX)
Method: Least Squares
Date: 08/27/16 Time: 13:43
Sample (adjusted): 2 2355
Included observations: 2354 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
LNEX(-1)	0.000130	0.000755	0.171718	0.8637
C	9.82E-05	0.000149	0.656895	0.5113

R-squared	0.000013	Mean dependent var	0.000113
Adjusted R-squared	-0.000413	S.D. dependent var	0.005926
S.E. of regression	0.005928	Akaike info criterion	-7.417544
Sum squared resid	0.082641	Schwarz criterion	-7.412647
Log likelihood	8732.449	Hannan-Quinn criter.	-7.415760
F-statistic	0.029487	Durbin-Watson stat	1.995578
Prob(F-statistic)	0.863674		

ADF: Trend and Intercept

Null Hypothesis: LNEX has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=26)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.026490	0.1251
Test critical values:	1% level	-3.961944	
	5% level	-3.411717	
	10% level	-3.127739	

*MacKinnon (1996) one-sided p-values.

ADF: None

Null Hypothesis: LNEX has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=26)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		0.672671	0.8611
Test critical values:	1% level	-2.565945	
	5% level	-1.940958	
	10% level	-1.616609	

*MacKinnon (1996) one-sided p-values.

Trend Stationary I

- a common practice to make a trending time series stationary is to remove the trend from it

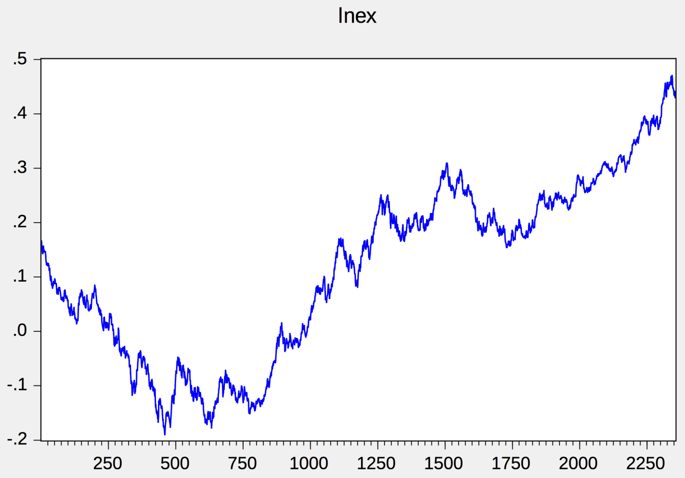
$$LEX_t = A_1 + A_2 t + v_t \tag{5}$$

- t is a time trend variable [1,2,...], and v_t error term with the usual properties
- after running the regression, we obtain

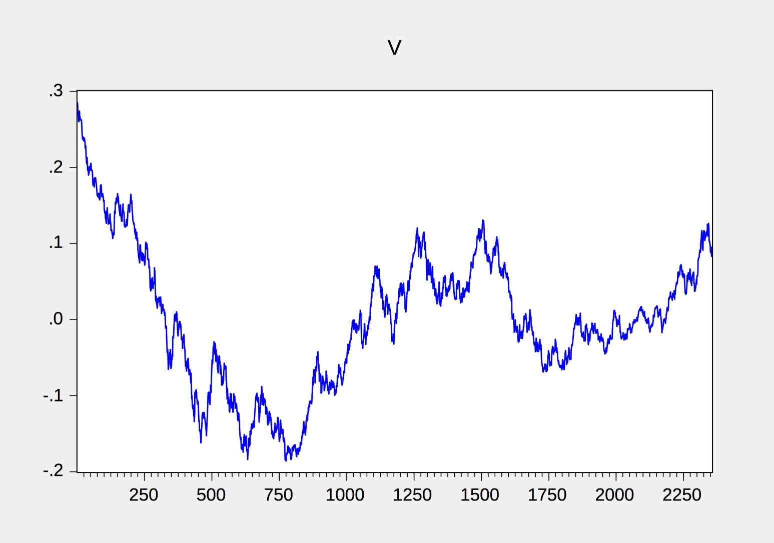
$$\hat{v} = LEX_t - a_1 - a_2 t \tag{6}$$

- $\hat{v}$ represents the detrended LEX time series

Exchange Rate Series



Trend Stationary II



Trend Stationary III

Null Hypothesis: LNEX has a unit root
Exogenous: None
Lag Length: 0 (Automatic - based on SIC, maxlag=26)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		0.672671	0.8611
Test critical values:	1% level	-2.565945	
	5% level	-1.940958	
	10% level	-1.616609	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: V has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=26)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.025477	0.0327
Test critical values:	1% level	-3.432933	
	5% level	-2.862567	
	10% level	-2.567362	

*MacKinnon (1996) one-sided p-values.

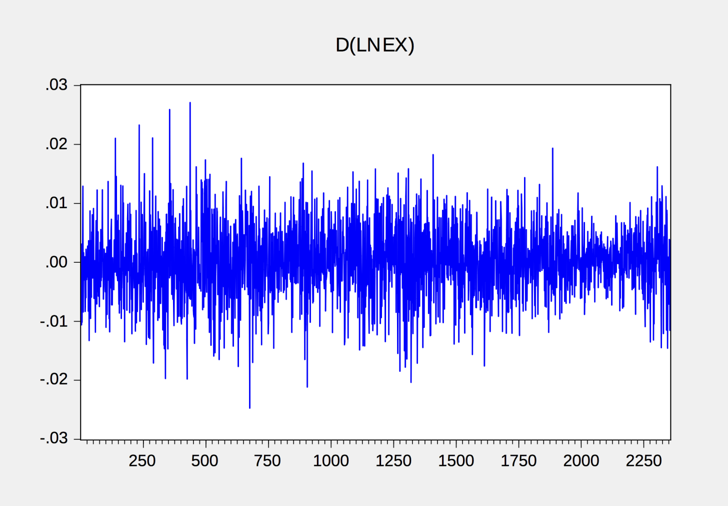
Trend Stationary IV

- a time series becomes stationary after detrend; trend stationary (stochastic) process TSP
- a process with a deterministic trend is nonstationary but not a unit root process

Difference Stationary I

- if a time series becomes stationary after taking its first difference, it is called a difference stationary (stochastic process) DSP

Difference Stationary II



Difference Stationary III

Date: 08/27/16 Time: 16:37
Sample: 1 2355
Included observations: 2354

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.002	0.002	0.0113	0.915
		2	-0.001	-0.001	0.0125	0.994
		3	-0.017	-0.017	0.6673	0.881
		4	0.051	0.052	6.9213	0.140
		5	-0.036	-0.037	10.017	0.075
		6	0.016	0.016	10.643	0.100
		7	0.020	0.022	11.582	0.115
		8	-0.024	-0.028	12.970	0.113
		9	0.003	0.008	12.997	0.163
		10	-0.013	-0.015	13.379	0.203
		11	-0.003	-0.004	13.396	0.268
		12	0.012	0.016	13.735	0.318
		13	0.034	0.030	16.482	0.224
		14	-0.003	-0.001	16.501	0.284
		15	-0.032	-0.031	18.857	0.220
		16	0.011	0.010	19.140	0.261
		17	0.002	0.000	19.149	0.320

Difference Stationary IV

Null Hypothesis: D(LNEX) has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=26)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-48.38514	0.0001
Test critical values:	1% level	-3.432934	
	5% level	-2.862568	
	10% level	-2.567362	

*MacKinnon (1996) one-sided p-values.

Integrated Time Series I

- a time series becomes stationary after differencing it once called integrated of order one $I(1)$
- if it has to be differenced twice to become stationary; integrated of order two $I(2)$
- if differenced d times to make it stationary, then it is integrated of order d; $I(d)$
- a stationary time series is integrated of order zero; $I(0)$

Integrated Time Series II

- an $I(0)$ series fluctuate around its mean with constant variance, while an $I(1)$ series meanders wildly
- i.e., an $I(0)$ series is mean reverting, whereas an $I(1)$ series does not show such a tendency
- an $I(1)$ series can drift away from the mean permanently - that is why said to have a stochastic trend
- the autocorrelations in a correlogram of an $I(0)$ series decline to zero very rapidly as the lag increases, while those of an $I(1)$ decline for slowly
- most nonstationary economic time series do not need to be differenced more than once or twice

Strictly Stationarity

- a time series is strictly stationary if all moments of its probability distribution and not just the first two (i.e., mean and variance) are invariant over time
- if, however, the stationary process is normal, the weakly stationary stochastic process is also strictly stationary, for the normal process is fully specified by its two moments, mean and variance

go back

Stochastic

- the term "stochastic" comes from the Greek word *stokhos*, which means a target or bullseye
  - anyone who throws darts at a dashboard knows that the process of hitting the bullseye is a random process
  - out of several darts, a few will hit the bullseye, but most of them will be spread around it in a random fashion

go back

Unit Root

$$LEX_t = \beta_1 + \beta_2 t + C\ LEX_{t-1} + u_t$$

- subtract LEX_{t-1} from both sides gives

$$(LEX_t - LEX_{t-1}) = \beta_1 + \beta_2 t + C\ LEX_{t-1} - LEX_{t-1} + u_t$$

$$\Delta LEX_t = \beta_1 + \beta_2 t + \beta_3 LEX_{t-1} + u_t$$

- where $\beta_3 = (C - 1)$
- if $C = 1$, β_3 will be zero, hence the name unit root

go back

White Noise

- a purely random, or white noise, time series
- such as a time series has
  - constant mean,
  - constant (i.e., homoscedastic) variance, and is
  - serially uncorrelated
  - its mean is often assumed to be zero
- the error term in the CLRM is assumed to be white noise (stochastic) process $u_t \sim iid(0, \sigma^2)$
  - independently, and identically distributed with zero mean and constant variance

go back