ES1004 Econometrics by Example

Lecture 8: Dynamic Regression Models

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Gujarati textbook, second edition [chapter 7]

6th August 2016

CLRM Assumptions

A_1: model is linear in parameters

A_2: regressors are fixed non-stochastic

A_3: the expected value of the error term is zero $E(u_i|X) = 0$

A_4: homoscedastic or constant variance of errors $var(u_i|X) = \sigma^2$

A_5: no autocorrelation, $cov(u_i, u_j) = 0, i \neq j$

A_6: no multicollinearity; no perfect linear relationships among the Xs

A_7: no specification bias

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Basic Idea I

- CLRM assumes the model is 'correctly' specified
    - there is no such thing as a perfect model
- an econometric model tries to capture the main features of an economic phenomenon
    - taking into account the underlying economic theory, prior empirical work, intuition, and research skills
- no model can take into account every single factor that affects a particular object of research

Basic Idea II

- a ‘correctly’ specified model . . .
  1. does not exclude any "core" variables
  2. does not include superfluous variables
  3. has the suitable functional form
  4. has no measurement errors

Basic Idea III

- a ‘correctly’ specified model . . .
  5. takes into account outliers in the data
  6. the probability distribution of the error term is well specified
  7. includes non-stochastic regressors
  8. no simultaneity bias

Equilibrium in Theory

- economic theory is often stated in `static` or `equilibrium` form
  - e.g. the equilibrium price of a commodity (or service) is determined by the intersection of the relevant demand and supply curves
- however, equilibrium is not determined instantaneously
  - i.e., a process of trial and error which takes time
- neglecting the dynamic [time] aspect may lead to a specification error

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Permanent Income Hypothesis

- Milton Friedman
  - current consumption is a function of permanent income
  - weighted average of quarterly income going back 16 quarters

$$Y_t = A + B_0 X_t + B_1 X_{t-1} + B_2 X_{t-2} + \cdots + B_1 6 X_{t-16} + u_t \quad (1)$$

- X_t income in the current period, X_{t-1} income lagged one quarter
- β weights attached to the income in the various quarters

Distributed Lag Model I

- current value of Y is affected by current and lagged values of X
- β_0
  - the short-run or impact multiplier
  - gives the change in the mean value of Y following a unit change in X in the same time period
- if the change in X is kept at the same level thereafter, $(\beta_0 + \beta_1)$ gives the change in mean Y in the next period
- the partial sums are called interim or intermediate multipliers

Distributed Lag Model II

- after k periods, we obtain

$$\sum_{0}^{k} \beta_k = \beta_0 + \beta_1 + \cdots + \beta_k \quad (2)$$

- known as the long run or total multiplier
- gives the ultimate change in mean consumption expenditure following a (sustained) unit increase in the income

Distributed Lag Model: An Example

$$Y_t = constant + 0.4X_t + 0.2X_{t-1} + 0.15X_{t-2} + 0.1X_{t-3}$$

- the impact multiplier 0.4, the interim multiplier 0.75, total or long-run multiplier 0.85
- if income increases by \$1000 in year t and assuming this increase is maintained, consumption will increase
  - \$400 in the first year
  - another \$200 in the second year
  - another \$150 in the third year
  - final total increase \$750
  - presumably, the consumer will save \$250

OLS Estimation

- we can estimate Eq. 1 by the usual OLS method
- however, this is not practical

1. how do we decide how many lagged terms we use
2. using several lagged terms leads to fewer degrees of freedom to do meaningful statistical analysis especially of the sample size is small
3. successive values of the lagged term are likely to be correlated causing multicollinarity and imprecise estimation of the regression coefficients

Koyck Distributed Lag Model I

- we can express Eq. 1 in a general form as

$$Y_t = A + B_0 X_t + B_1 X_{t-1} + B_2 X_{t-2} + \cdots + u_t \quad (3)$$

- infinite DLM as we have not defined the length of the lag [how far back in time we want to travel]
- note that Eq. 1 represents a finite DLM with 16 lagged terms

Koyck Distributed Lag Model II

- Koyck uses the geometric probability distribution to estimate the parameters of Eq. 3
- assuming that all the β coefficients in Eq. 3 have the same sign, Koyck assumed that they decline geometrically as follows

$$B_k = \beta_0 \lambda^k, \quad k = 0, 1, \ldots; \quad 0 < \lambda < 1 \tag{4}$$

  - λ the rate of decline of decay
  - $(1 - \lambda)$ the speed of adjustment [i.e., how fast consumption expenditure adjusts to the new income level]

Koyck Distributed Lag Model III

- the value of β_k in Eq. 4 depends on β_0 and λ
- a value of λ close to 1
  - β_k declines slowly [i.e., X values in distant past will have some impact on the current value of Y]
- a value of λ close to zero
  - the impact of X in the distant past will have little impact on the current Y
- assuming $0 < \lambda < 1$ means that
  - each successive β coefficient is numerically smaller than each preceding β
  - as we go back into the distant past, the effect of that lag on Y becomes progressively smaller

Koyck DLM Estimation

$$Y_t = A + B_0 X_t + B_0 \lambda X_{t-1} + B_0 \lambda^2 X_{t-2} + B_0 \lambda^3 X_{t-3} + \cdots + u_t \tag{5}$$

- not easy to estimate an infinite number of coefficients and the adjustment coefficient λ enters highly nonlinearly
- Koyck transformation

$$Y_t = A(1-\lambda) + \beta_0 X_t + \lambda Y_{t-1} + v_t \tag{6}$$

  - where $v_t = u_t - \lambda u_{t-1}$

Koyck transformation

$$Y_t = A(1-\lambda) + \beta_0 X_t + \lambda Y_{t-1} + v_t$$

- not that the lagged value of the dependent variable appears as a regressor [autoregressive model]
- instead of estimating an infinite number of parameters in Eq.3, we estimate only three parameters in model 6

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Autoregressive Model

$$Y_t = A(1-\lambda) + \beta_0 X_t + \lambda Y_{t-1} + v_t$$

- the impact of a unit change in X on the mean value of Y
  - the short-run impact $\rightarrow$ the coefficient of X, β_0
  - the long-run impact $\rightarrow$ $\beta_0/(1-\lambda)$
  - since λ lies between 0 and 1, the long-run impact will be greater than the short-run impact [it takes time to adjust to the changed income]

Autoregressive Model: Estimation I

$$Y_t = A(1-\lambda) + \beta_0 X_t + \lambda Y_{t-1} + v_t$$

- estimating Eq. 6 poses formidable challenges

1. even if u_t satisfies the classical assumptions, the composite error term v_t may not [will be serially correlated]
2. since Y_t is stochastic, Y_{t-1} will be stochastic too
   - OLS assumes regressors to be either non-stochastic, or if stochastic, they must be distributed independently of the error term
   - Y_{t-1} and v_t are correlated $\rightarrow$ OLS estimators are not even consistent
3. we cannot use Durbin-Watson to test for serial correlation

Autoregressive Model: Estimation II

$$Y_t = A(1-\lambda) + \beta_0 X_t + \lambda Y_{t-1} + v_t$$

- the Koyck model, although elegant, poses formidable estimation problems
- the error term v_t is autocorrelated but we can use HAC standard errors discussed in lecture 6
- the more series problem is correlation between the lagged Y_t and the error term v_t
  - find a proxy for Y_{t-1} which is highly correlated with Y_{t-1} and yet uncorrelated with v_t [an instrumental variable IV]

Personal Consumption Expenditure: OLS Estimation

Dependent Variable: PCE
Method: Least Squares
Sample (adjusted): 1961 2009
Included Observations: 49 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	−485.8849	197.5245	−2.459872	0.0177
DPI	0.432575	0.081641	5.298529	0.0000
PCE(−1)	0.559023	0.084317	6.630052	0.0000

R-squared	0.998251	Mean dependent var	19602.16
Adjusted R-squared	0.998175	S.D. dependent var	6299.838
S.E. of regression	269.1558	Akaike info criterion	14.08773
Sum squared resid	3332462	Schwarz criterion	14.20355
Log likelihood	−342.1493	Hannan–Quinn criter.	14.13167
F-statistic	13125.09	Durbin–Watson stat	0.708175
Prob(F-statistic)	0.000000		

Personal Consumption Expenditure: HAC

Dependent Variable: PCE
Method: Least Squares
Sample (adjusted): 1961 2009
Included Observations: 49 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey–West fixed bandwidth = 4.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	−485.8849	267.7614	−1.814619	0.0761
DPI	0.432575	0.098339	4.398823	0.0001
PCE(−1)	0.559023	0.102057	5.477587	0.0000

R-squared	0.998251	Mean dependent var	19602.16
Adjusted R-squared	0.998175	S.D. dependent var	6299.838
S.E. of regression	269.1558	Akaike info criterion	14.08773
Sum squared resid	3332462	Schwarz criterion	14.20355
Log likelihood	−342.1493	Hannan–Quinn criter.	14.13167
F-statistic	13125.09	Durbin–Watson stat	0.708175
Prob(F-statistic)	0.000000		

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Personal Consumption Expenditure: Proxy

Dependent Variable: PCE
Method: Least Squares
Sample (adjusted): 1961 2009
Included Observations: 49 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey–West fixed bandwidth = 4.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	−1425.511	372.3686	−3.828224	0.0004
DPI	0.934361	0.175986	5.309287	0.0000
DPI(−1)	0.038213	0.177358	0.215455	0.8304

R-squared	0.996583	Mean dependent var	19602.16
Adjusted R-squared	0.996434	S.D. dependent var	6299.838
S.E. of regression	376.1941	Akaike info criterion	14.75736
Sum squared resid	6510013	Schwarz criterion	14.87318
Log likelihood	−358.5553	Hannan–Quinn criter.	14.80130
F-statistic	6707.481	Durbin–Watson stat	0.351356
Prob(F-statistic)	0.000000		

ARDL Model I

$$
\begin{aligned}
Y_t = A_0 + A_1 Y_{t-1} + A_2 Y_{t-2} + \cdots + A_p Y_{t-p} \\
+ \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \cdots + \beta_q X_{t-q} + u_t
\end{aligned}
$$

- the lagged Ys constitute the autoregressive part
- the lagged Xs constitute the distributed part
- p autoregressive terms and q distributed lag terms

ARDL Model II

- advantages of an ARDL model
  - captures the dynamic effects of the lagged Ys and also those of the lagged Xs
  - can eliminate autocorrelation in the error term if sufficient number of lags of both variables included
  - are often used for forecasting and estimating the multiplier effects of the regressors in the model

ARDL: Consumption Function

$$Y_t = A_0 + A_1 Y_{t-1} + \beta_0 X_t + \beta_1 X_{t-1} + u_t, \quad A_1 < 1 \tag{7}$$

- $Y_t = PCE$ and $X = DPI$
- ARDL (1,1) model [Eq. 7] enables us to find the dynamic effects of a change in DPI on current value and future values of PCE
- the immediate effect [impact multiplier] of a unit change in DPI given by β_0
- if the unit change in DPI is sustained, the long-run multiplier is given by $\frac{\beta_0+\beta_1}{1-A_1}$

ARDL: Assumptions

- we have to make certain assumptions
  1. the variables X and Y are stationary
  2. the expected mean value of the error term u_t is zero
  3. the error term is serially uncorrelated
  4. the X variables are exogenous - at least weakly so [uncorrelated with the error term]

ARDL: OLS estimation

Dependent Variable: PCE
Method: Least Squares
Sample (adjusted): 1961 2009
Included Observations: 49 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	−281.2019	161.0712	−1.745823	0.0877
DPI	0.824591	0.097977	8.416208	0.0000
PCE(−1)	0.805356	0.081229	9.914632	0.0000
DPI(−1)	−0.632942	0.118864	−5.324935	0.0000

R-squared	0.998927	Mean dependent var	19602.16
Adjusted R-squared	0.998855	S.D. dependent var	6299.838
S.E. of regression	213.1415	Akaike info criterion	13.63990
Sum squared resid	2044318.	Schwarz criterion	13.79433
Log likelihood	−330.1775	Hannan−Quinn criter.	13.69849
F-statistic	13962.93	Durbin−Watson stat	1.841939
Prob(F-statistic)	0.000000		

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ARDL: HAC

Dependent Variable: PCE
Method: Least Squares
Sample (adjusted): 1961 2009
Included Observations: 49 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey–West fixed bandwidth = 4.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	−281.2019	117.3088	−2.397107	0.0207
PCE(−1)	0.805356	0.071968	11.19044	0.0000
DPI	0.824591	0.114989	7.171026	0.0000
DPI(−1)	−0.632942	0.119717	−5.286977	0.0000

R-squared	0.998927	Mean dependent var	19602.16
Adjusted R-squared	0.998855	S.D. dependent var	6299.838
S.E. of regression	213.1415	Akaike info criterion	13.63990
Sum squared resid	2044318.	Schwarz criterion	13.79433
Log likelihood	−330.1775	Hannan–Quinn criter.	13.69849
F-statistic	13962.93	Durbin–Watson stat	1.841939
Prob(F-statistic)	0.000000		

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