ES1004 Econometrics by Example

Lecture 4: Multicollinearity

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Gujarati textbook, second edition

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CLRM Assumptions

A$_1$: model is linear in parameters

A$_2$: regressors are fixed non-stochastic

A$_3$: the expected value of the error term is zero $E(u_i|X) = 0$

A$_4$: homoscedastic or constant variance of errors $var(u_i|X) = \sigma^2$

A$_5$: no autocorrelation, $cov(u_i, u_j) = 0, i \neq j$

A$_6$: no multicollinearity; no perfect linear relationships among the Xs

A$_7$: no specification bias

Basic Idea

- CLRM assumes no exact linear relationship among explanatory variables A_6
- perfect multicollinearity
  - an exact relationship amongst the x's
  - is rarely encountered in practice, unless as a result of 'specification error' e.g., dummy variable trap
- imperfect multicollinearity
  - when explanatory variables are highly correlated
  - is a matter of degree
  - typically in macroeconomic time series data

Perfect Multicollinearity I

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \cdots + \beta_k X_{ki} + u_i \tag{1}$$

- if, for example, $X_{2i} + 3X_{3i} = 1$ we have perfect collinearity for $X_{2i} = 1 - 3X_{3i}$
- then we cannot include both X_{2i} and X_{3i} in the same regression model
- we cannot estimate the regression coefficients

Perfect Multicollinearity II

- examples of perfect collinearity
  - if we introduce income variables in both dollars and cents in the consumption function
  - dummy variable trap: when including as many dummies as the number of groups with the presence of the intercept
- in practice, exact linear relationships among regressors is a rarity

Imperfect Multicollinearity

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \cdots + \beta_k X_{ki} + u_i$$

- if we have $X_{2i} + 3X_{3i} + v_i = 1$ where v_i is a random term, for $X_{2i} = 1 - 3X_{3i} - v_i$
- then we have imperfect multicollinearity
- no perfect linear relationship between the two variables
- in most cases, you we deal with imperfect (or near) collinearity rather than perfect collinearity

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Multicollinearity and OLS Estimation

- OLS estimators still BLUE
- high R^2 but will have insignificant coefficients
- regression coefficients are very sensitive to small changes in the data, especially of the sample is relatively small

- if two variables are highly collinear it is very difficult to isolate the impact of each variable separately on the regressand

Modelling Expenditure: Data

Expenditure ($)	Income ($)	Wealth ($)
70	80	810
65	100	1009
90	120	1273
95	140	1425
110	160	1633
115	180	1876
120	200	2052
140	220	2201
155	340	2435
150	260	2686

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Modelling Expenditure: Estimation

Dependent variable	Intercept	Income	Wealth	R^2
Expenditure	24.7747	0.9415	−0.0424	0.9635
	(3.6690)	(1.1442)	(−0.5261)	
Expenditure	24.4545	0.5091	–	0.9621
	(3.8128)	(14.2432)		
Expenditure	24.4410	–	0.0498	0.9567
	(3.5510)	–	(13.2900)	
Wealth	7.5454	10.1909	–	0.9979
	(0.2560)	(62.0405)		

Testing for Collinearity

- there is no unique test for multicollinearity

1. high R^2 but few significant t ratios
2. high pairwise correlations among explanatory variables
3. high partial coefficients
4. significant F-test for auxiliary regressions
5. high variance inflation factor [low tolerance factor]

Married Women's Hours of Work: Data

- Mroz (1987) Econometrica, 55, 765-99
- assessing the impact of several socio-economic variables
- data in Table 4.4 [see Piazza]
- cross-sectional data on 753 married women in 1975
- 325 married women did not work [i.e., zero hours of work]

Married Women's Hours of Work: Variables I

- hours ☞ hours worked in 1975 [dependent variable]
- age ☞ woman's age in years
- educ ☞ years of schooling
- exper ☞ actual labour market experience
- faminc ☞ family income in 1975
- fathereduc ☞ father's years of schooling
- hage ☞ husband's age
- heduc ☞ husband's years of schooling

Married Women's Hours of Work: Variables II

- hhours ☞ hours worked by husband
- hwage ☞ husband's hourly wage, 1975
- kids618 ☞ number of kids between ages 6 and 18
- kidsl6 ☞ number of kids under age 6
- wage ☞ estimated wage from earnings
- mothereduc ☞ mother's years of education
- mtr ☞ marginal tax rate facing a woman
- unemployment ☞ unemployment rate in county of residence

Married Women's Hours of Work: A priori

- we would expect a
  - positive sign ☞ education, experience, father's education, mother's education
  - negative sign ☞ age, husband's age, husband's hours of work, husband's wage, marginal tax rate, unemployment rate, number of kids under 6

Estimation

Dependent Variable: HOURS
Method: Least Squares
Date: 05/20/16 Time: 09:44
Sample: 1 753 IF HOURS>0
Included observations: 428

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	8595.360	1027.190	8.367843	0.0000
AGE	-14.30741	9.660582	-1.481009	0.1394
EDUC	-18.39847	19.34225	-0.951206	0.3421
EXPER	22.88057	4.777417	4.789318	0.0000
FAMINC	0.013887	0.006042	2.298543	0.0220
FATHEREDUC	-7.471448	11.19227	-0.667554	0.5048
HAGE	-5.586216	8.938425	-0.624966	0.5323
HEDUC	-6.769259	13.98780	-0.483940	0.6287
HHOURS	-0.473547	0.073274	-6.462701	0.0000
HWAGE	-141.7821	16.61801	-8.531837	0.0000
KIDS618	-24.50866	28.06160	-0.873388	0.3830
KIDSL6	-191.5649	87.83197	-2.181038	0.0297
WAGE	-48.14963	10.41198	-4.624447	0.0000
MOTHEREDUC	-1.837597	11.90008	-0.154419	0.8774
MTR	-6272.598	1085.438	-5.778864	0.0000
UNEMPLOYMENT	-16.11532	10.63729	-1.514984	0.1305

R-squared	0.339159	Mean dependent var	1302.930
Adjusted R-squared	0.315100	S.D. dependent var	776.2744
S.E. of regression	642.4347	Akaike info criterion	15.80507
Sum squared resid	1.70E+08	Schwarz criterion	15.95682
Log likelihood	-3366.286	Hannan-Quinn criter.	15.86500
F-statistic	14.09655	Durbin-Watson stat	2.072493
Prob(F-statistic)	0.000000		

Dependent Variable and Sample

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Insignificant Coefficients

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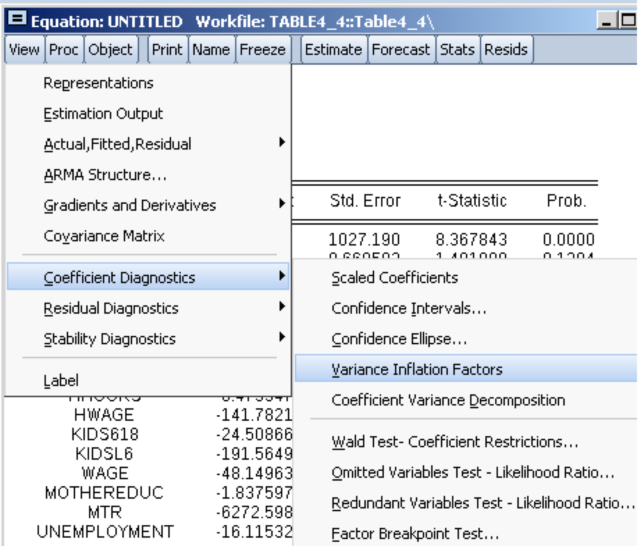
Coefficient of Determination R^2

R-squared	0.339159	Mean dependent var	1302.930
Adjusted R-squared	0.315100	S.D. dependent var	776.2744
S.E. of regression	642.4347	Akaike info criterion	15.80507
Sum squared resid	1.70E+08	Schwarz criterion	15.95682
Log likelihood	-3366.286	Hannan-Quinn criter.	15.86500
F-statistic	14.09655	Durbin-Watson stat	2.072493
Prob(F-statistic)	0.000000		

Variance Inflation Factor VIF



Variance Inflation Factor VIF

Variable	Coefficient Variance	Uncentered VIF	Centered VIF
C	1055118.	1094.176	NA
AGE	93.32684	176.2509	5.756163
EDUC	374.1226	64.19296	2.021618
EXPER	22.82372	5.555480	1.532452
FAMINC	3.65E-05	27.18584	5.144349
FATHEREDUC	125.2668	12.10382	1.608908
HAGE	79.89544	170.1046	5.224349
HEDUC	195.6586	34.13956	1.864803
HHOURS	0.005369	29.66169	1.887424
HWAGE	276.1581	18.59817	3.643849
KIDS618	787.4534	2.900083	1.410795
KIDSL6	7714.456	1.383181	1.225962
WAGE	108.4093	3.191149	1.229041
MOTHEREDUC	141.6118	14.90258	1.603344
MTR	1178175.	552.9496	7.215127
UNEMPLOYMENT	113.1520	9.646116	1.077137

How to Remedy for Collinearity

- what should we do when there is multicollinearity
  - nothing, for we often have no control over the data
  - redefine the model by excluding variables may attenuate the problem
  - cautious needed as to no omit relevant variables
- principal components analysis
  - construct artificial variables from regressors such that they are orthogonal to one another
  - these principal components becomes the regressors in the model
  - yet, the interpretation of the coefficients is not straightforward

Revised Women's Hours of Work

Dependent Variable: HOURS
Method: Least Squares
Date: 05/21/16 Time: 16:17
Sample: 1 753 IF HOURS>0
Included observations: 428

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	8484.524	987.5952	8.591094	0.0000
AGE	-17.72740	4.903114	-3.615540	0.0003
EDUC	-27.03403	15.79456	-1.711604	0.0877
EXPER	24.20345	4.653332	5.201315	0.0000
FAMINC	0.013781	0.005866	2.349213	0.0193
HHOURS	-0.486474	0.070462	-6.904046	0.0000
HWAGE	-144.9734	15.88407	-9.126972	0.0000
KIDSL6	-180.4415	86.36960	-2.089178	0.0373
WAGE	-47.43286	10.30926	-4.600995	0.0000
MTR	-6351.293	1029.837	-6.167278	0.0000
UNEMPLOYMENT	-16.50367	10.55941	-1.562935	0.1188